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GEORGE WU
HANS LIEBLER

Canonical Decision Problems

It is an understatement to say that managers face many different types of decision problems. This set of decision vignettes focuses on *canonical* decision problems. *Webster's New Collegiate Dictionary* defines canonical as "reduced to the simplest or clearest schema possible." In this spirit, these vignettes present simplified versions of some "standard" problems in management, basic problems that arise in surprising frequency. While we have simplified the decision situations, we have tried to preserve the "essence" of the problems. At the end of the exercises, we include some technical exhibits which suggest approaches for dealing with more complex versions of these problems, as well as some practical suggestions on how to obtain information necessary for analyzing these situations.

1. Product Development Sequencing

Eldorado Electronics is under contract to make 10,000 medical scanners. Eldorado must decide whether to make or buy the circuit board to which it will attach its proprietary microchip. There are two stages in making a circuit board, development and fabrication. The chance that the Eldorado's engineers will succeed in development is 90%. The cost of development are $50,000. The fabrication stage has an 80% chance of success and will cost $30,000. If both stages are successful, the variable cost of producing a circuit board will be $9/board. The development and fabrication processes use different sets of engineers and can proceed independently. If either stage is unsuccessful, Eldorado must purchase circuit boards from an outside vendor for a cost of $24/circuit board.

The scanners are scheduled for delivery in 8 months. The development and fabrication stages each take 3 months, and assembly of the finished product takes an additional 2 months. Purchasing the circuit boards from an outside source requires a lead time of 3 months, although for a premium of $10/board, Eldorado can purchase boards for immediate delivery.

a) Suppose that Eldorado attempts to make the board in-house. How should Eldorado proceed if the goal is to minimize expected costs?

b) Should Eldorado purchase the circuit boards outside or attempt to make the board in-house?

Professor George Wu and Research Associate Hans Liebler prepared these vignettes.

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2. Options for flexibility

Susan Smart, a recent graduate of MIT, has decided to start her own business, Frill Less Internet Provider (FLIP). She is offering small businesses and residential customers Internet access, providing a fast and convenient dial-up connection by modem. To start this business she must buy an Internet server, a special computer designed to bundle all incoming calls and establish the connection to the Internet backbone system. She is currently evaluating two different offers from the computer manufacturer Xentec, one for a X100 server for a price of $14,000 and another for a X120 server priced at $22,000. The most important factor for her investment decision is the peak number of subscribers one server can handle. A rule of thumb is that the peak capacity of the server should be 10% of the total number of subscribers. The number of subscribers Susan can take is constrained by the capacity of her server. The X100 has a capacity of 80 connections at peak level; the X120 can handle up to 140 connections. Thus, buying the X100 permits FLIP to take up to 800 subscribers, while the X120 allows a maximum of 1,400 customers.

The problem is that Susan does not know how many customers she will attract with her new service. She believes that it is equally likely that she will attract 400 or 800 subscribers during the first year. If the demand is high in the first year she believes that there is a 50% chance of getting 1,200 subscribers during the second year and a 50% chance that demand will stay at 800. If the demand is low during the first year, there is a 50% chance that it will remain at 400 and a 50% chance that it will go up to 800.

Susan plans to charge a $20 flat monthly fee to her subscribers, independent of the hours of usage. The variable costs of setting up the modem connection and monitoring the activities are expected to be $9 per month and customer. The total overhead expenses (salaries, office rent, etc.) are expected to be $6,000 per month.

a) Should she buy one X100 or one X120 server? Base your decision on the expected profit over the following two years. Also assume for simplicity that the servers have a useful life of 2 years and no salvage value. Finally disregard time value of money and taxes.

Option to Expand

b) Susan expects that used Xentec servers will be available at 40% of the original price one year from now. Does her initial investment decision change if she can buy a second used X100 server after one year?

Option to Switch

c) Instead of buying another X100 server, she might want to sell her X100 after one year and buy a used X120 instead. Are there any circumstances in which Susan would exercise this option? What is the expected monetary value of this option?

Option to Abandon

d) If the demand turns out to be low after one year of operation, Susan might want to abandon the entire business. If she abandons the business, she can sell either the X120 computer or the X100 at 40% of the original price. How does this option to abandon affect the initial investment decision?

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Option to Wait

e) As a result of all the uncertainty, Susan is considering delaying the whole investment for one year. If the market looks favorable, then she will buy a used server. Is this alternative better then the options described before?

3. Market Research

You are brand manager at Arrow Cosmetics and responsible for Silkskin™ body lotion. Silkskin's annual contribution is currently $2.1 million. The R&D department has just developed a new Aloe Vera-based body lotion formula. You must decide whether you want to change the formula for the Silkskin™ lotion or stay with the old one.

The variable costs will be slightly higher for the new formula ($7.00 per bottle), but you guess that an increase in sales might compensate for the higher costs. You plan on selling the new Silkskin™ for $12.00 per bottle. You can imagine three annual sales scenarios: 600,000, 300,000 and 200,000 units sold, which you estimate the chances to be .30, .40, and .30, respectively. See **Exhibit 1** for a detailed description of how these probabilities might be obtained in practice.

a) Under which scenario(s) would you introduce the new Silkskin™?

b) Would you introduce the new formula?

c) Imagine a market research firm that provides perfect information. In other words, if the firm tells you that annual sales will be "x", then sales will in fact be x. What is the maximum price you would be willing to pay for the services of such a company?

Marble Research

You are considering hiring Marble Research, one of the nation's leading market research companies. For $150,000, they will perform a test market study on your new formula. Although Marble can't predict sales figures exactly, they will indicate whether they believe the product will be a "Hit" or a "Flop." You estimate that Marble will indicate "Hit" and "Flop" with equal likelihood.

If Marble tells you that the formula is going to be a "Hit," then you believe that the probability of selling 600,000, 300,000 or 200,000 units will be .60, .30, and .10 respectively. If they indicate "Flop," the probabilities of selling 600,000, 300,000 or 200,000 units will be 0, .50, and .50 respectively. See **Exhibit 2** for a detailed description of how these probabilities might be obtained in practice.

d) Would you go forward if Marble indicates "Hit"?

e) Would you introduce the new formula if Marble indicates that the new Silkskin™ will "flop"?

f) Would you hire Marble? If no, suppose you try to negotiate a discount with Marble. What is the maximum price you are willing to pay for the information they are providing?

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Bargain Research Consortium

The Bargain Research Consortium (BRC), one of Marble's competitors, is offering you a similar market study for just $15,000. They will predict a "Hit" with probability .4, in which case the probabilities of selling 600,000, 300,000 and 200,000 units are .45, .40, and .15 respectively. If they predict a "Flop," the probabilities of selling 600,000, 300,000, and 200,000 units will be 20%, 40% and 40%.

g) Would you hire BRC?

h) What is the most you would pay BRC's forecast?

4. Litigation Decision[1]

Sarah Gable is the President of Gable's Department Store. Last year Jeremiah Walls, a 35 year-old man, fell on the ice in the Gable department store parking lot. Walls, who hurt his wrist and fractured his ankle in the accident, has brought a suit against Gable's.

Of course, Sarah Gable would like to settle out of court and hence has initiated discussion with Walls' counsel. However, Gable is willing to go to trial if Walls' attorney is unreasonable. The first step of the process is a summary judgment in which the judge will decide whether to hear the case. It will cost Gable $3,000 in attorney fees to prepare for summary judgment, and she estimates that there is 90% chance that the judge will decide to hear the case and a 10% chance that the case will be dismissed. If the case is heard, Gable's best guess is that there's a 2-in-3 chance that Walls will win, with an expected award of $50,000. A court trial will cost Gable another $10,000 in lawyer's fees.

a) Assume that Gable makes her decisions on the basis of expected value. What is the most she would be willing to pay to avoid going to trial?

b) If Walls wins, Gable may appeal the decision. There is a 25% chance that the judge will hear the appeal. If the judge hears an appeal, there is a 90% chance that Walls will win, with the same expected award of $50,000. If Gable appeals, additional attorney fees will be $2,000, with an additional $5,000 if the appeal is heard. If Walls wins, should Gable appeal?

5. Bidding Decisions[2]

You are to submit a sealed bid for the rights to drill for oil in a large tract of ocean off the coast of California. The bidder who submits the highest sealed bid will be awarded the rights to drill. The owner of the tract (the U.S. government) has published geological information about the site, and experts generally agree there is a 20% chance of finding oil. In addition to purchasing the drilling rights, a buyer would have to spend approximately $10 million making exploratory drill holes in a search for oil. If oil is found, the estimated value of the tract would jump to about $150 million. (The owner would either develop the field or sell it to someone else.) If no oil is found, the drilling rights become worthless.

[1] Some of the estimates in this example are from Michael deCourcy Hinds, "Compiling Data, and Giving Odds, on Jury Awards," *The New York Times*, January 21, 1994.

[2] This example is taken from David E. Bell, *Bidding Exercises* (191-133).

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There is only one other serious bidder for this tract. You have studied their past bidding tactics and have concluded that there is a 25% chance they will submit a "low" bid of about $5 million, a 50% chance they will submit a "medium" bid of $10 million, and a 25% chance they will submit a "high" bid of $15 million.

Assume you represent a large company to whom this is a routine decision (it has no long-term strategic implications, nor is it viewed as especially risky).

a) Suppose you bid $16 million, thus ensuring that you win the rights to drill. What is the Expected Monetary Value of such a bid?

b) Assume that bids may be made only in $1 million increments. What is the appropriate bid to make? In the case of a tie, the winner is selected by a coin flip.

c) The three-point probability assessment is obviously simplistic. How might you proceed if you were to make a more refined assessment of the other party's bidding behavior in order to decide how much to bid?

6. Inventory Decisions[3]

With just three weeks to go before the start of the Charles River Jazz Festival, Buzz Ward, the festival's director, has found yet another way to capitalize on the event's enormous artistic success: sell compact disks of the early festival performances during later performances of the festival. Compact disk (CD) technology has progressed so rapidly within the past 10 years, and it is now possible to master and press CDs overnight. As novel as the idea seems, Buzz recognizes that such a project is extremely risky. To his knowledge, no other festival has undertaken such a project. Furthermore, if customers are unreceptive to the idea, the festival will be stuck with pile of unsold CDs.

For 12 years, every August, Cambridge hosts the Charles River Jazz Festival. The event spans three days, with performances on Friday night, and Saturday and Sunday afternoons. Each day requires a separate admission charge of $10. As Buzz envisions it, he will arrange first to have Friday night's performances recorded. Later that same evening, a compact disk will be pressed of that day's best performances; copies will be sold on Saturday and Sunday.

Comco, the local CD manufacturer, has quoted the following prices, which includes a sizable premium reflecting the features peculiar to this project:

Fixed Costs	
Production of master disk	$4,200
Machine set-up	$2,800
Unit costs per disk	
Packaging	$0.50
Other manufacturing costs	$2.50

[3] This example is a simplified version of George Wu, *Charles River Jazz Festival* (893-004).

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In addition, for each disk sold, CRJF will have to pay $3.00 in royalties to the artists involved (royalties are paid when a disk is sold, not pressed). If they go ahead with the project, CRJF will incur $8,000 in miscellaneous fixed costs (e.g., for promotion, additional vendors, sound engineers, etc.).

Buzz plans to sell each disk for $14. CRJF has been unable to negotiate an acceptable agreement with a record distributor to purchase unsold CDs. As a result, Buzz has decided that any unsold disks will be destroyed after the festival.

Since the Charles River Jazz Festival has never marketed CDs before, it is difficult for Buzz to estimate how many disks will be sold. Demand for CDs depends first and foremost on festival attendance, which in the past has varied substantially—from 3,300 to 30,600 per festival day. However, even if Buzz could predict attendance perfectly, it is still hard to predict CD sales, as it is unclear how receptive customers will be to such a new idea.

In spite of these difficulties, Buzz needs to estimate demand. He feels there is a 2/3 chance that Saturday demand will be 1,000 and a 1/3 chance that Saturday demand will be 3,000. If Saturday demand is only 1,000, it probably indicates that customers are unreceptive to the idea. If this is the case, low Sunday demand is also likely: the probabilities of Sunday demand of 1,000 and 3,000 will be .75 and .25 respectively. If Saturday demand is 3,000, Buzz believes that the probabilities of Sunday demand of 1,000 and 3,000 units will be .25 and .75 respectively.

a) Using Buzz's probabilities estimate, what is the probability that demand for compact disks will be exactly 4,000 CDs during the festival, i.e., over Saturday and Sunday combined?

b) Draw an appropriate decision tree for Buzz's problem, i.e., to determine how many CDs CRJF should press on Friday. Be sure to include probabilities and the value of endpoints.

c) How many CDs should Charles River Jazz Festival press on Friday? Use maximization of Expected Monetary Value as the criterion. What is the Expected Monetary Value of the best plan?

d) Suppose that Buzz could obtain information about the demand for CDs prior to committing to the CD project (i.e., before paying any fees to Comco or incurring any of the miscellaneous cost). What is the most he should pay for perfect information about the two-day demand for CDs?

e) Buzz has found a local record distributor who is willing to purchase unsold compact disks. How high a price would the record distributor have to pay, per disk, for Buzz to change his production quantity decision?

7. Who's Right? "Betting" on Theories[4]

Alison Silk had just been promoted to brand manager for Nopane, a mature drug product which had been marketed for almost a decade. After a careful analysis of the historic performance of the product, she became convinced that sales could be increased by repositioning the brand. The brand's advertising agency prepared a series of commercials that would shift the brand from the "emotional" approach pursued to date to a more "rational" appeal.

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[4] This example is a simplified version of David E. Bell, *Nopane Advertising Strategy* (893-005).

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To find out whether emotional copy would be successful in a national roll-out of the new strategy, she ran a market test in 24 randomly selected sales territories in the United States, testing the effect of the "emotional" campaign in 12 territories, the remaining 12 applied the traditional "rational" approach. Expressed in sales per 100 "prospects" (potential customers), Silk found that average sales for "rational" copy was 2.13 higher than average sales for "emotional" copy.

One week later, Alison presented her conclusions to Stanley Skamarycz, the Division Vice President whose approval would be needed for a change in advertising strategy. Alison showed him the experimental data. She was surprised by his response. "These results are worthless!" he said, "It's clear that our competitors just increased advertising expenditures in some of our test markets to run interference on your experiment. I'll be prepared to bet that no matter which copy we choose, they'll spend at the rate they've always spent."

Back in her office, Silk ran a regression that showed that the amount of competitive advertising was higher in sales territories in which Nopane had used "emotional" advertising strategy. Silk ran a second regression and found that average sales for "emotional" copy was 3.00 sales units higher than average sales for "rational" copy, controlling for competitive response.

Silk was now thoroughly confused. If competition reacted to the experiment as they would for a national rollout, as Silk thought they would, then "rational" was 2.13 sales units better than "emotional." However, if Skamarycz was right and competition would return to their old behavior, then "emotional" was 3.00 higher than "rational."

a) What should you do if Silk was right?

b) What should you do if Skamarycz was right?

c) Which advertising copy should Nopane use in the national rollout? Can you answer this question on the basis of the data provided? If yes, how? If no, how would you go about answering it?

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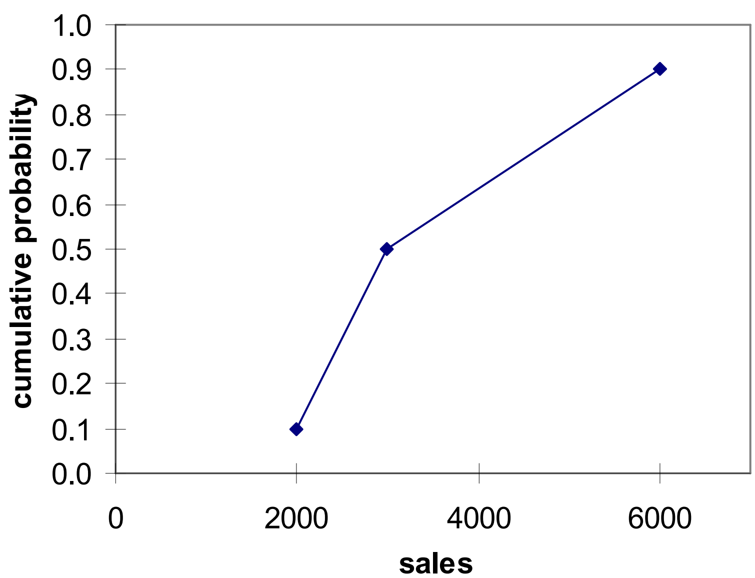
Exhibit 1 Probability Assessments for Continuing Quantities

In many situations, the critical uncertainty is a continuous quantity, such as demand or the price of IBM stock. In other words, it is possible that sales next year might be 10,000, 10,001, 10,002, etc. Of course, it is impossible to consider every possible sales level—analysis would simply be too complicated. Thus, we need to balance the need for simplification with the need to capture enough useful information about the uncertain quantity.1.

Three-point Approximations

You, the brand manager of Arrow Cosmetics, must decide whether to introduce the new product. You require information about how many units the new product will sell. Suppose you are interested in modeling three scenarios: low, medium, and high. You start by making some probabilistic estimates about the extreme cases. Your best guess is that there is only a .10 chance that sales will be below 2,000. In other words, there is a .90 chance that sales will be above 2,000. The number 2,000 is called the *.10 fractile*. Furthermore, you estimate that there is only a .10 chance that sales will be above 6,000 units (hence a .90 chance that sales be below 6,000). Thus, the .90 fractile is 6,000. Finally, you estimate that the median, or .50 fractile, is 3,000. In other words, you feel that it equally likely that sales will be above or below 3,000. The cumulative probability distribution is shown in **Figure 1**.

Figure 1



If you want to approximate the entire distribution with these points alone, an approach that is often used in practice is to assign .30 probability to the .10 fractile, .40 probability to the .50 fractile, and .30 probability to the .90 fractile. Why use .30, .40, and .30 as weights? Unfortunately, the answer to this question is beyond this note. It turns out that .30, .40, and .30 work well in approximating many standard probability distributions, including symmetric distributions such the normal.

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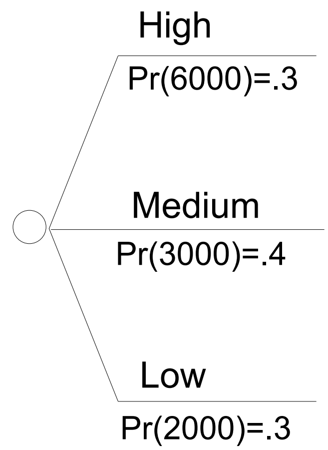
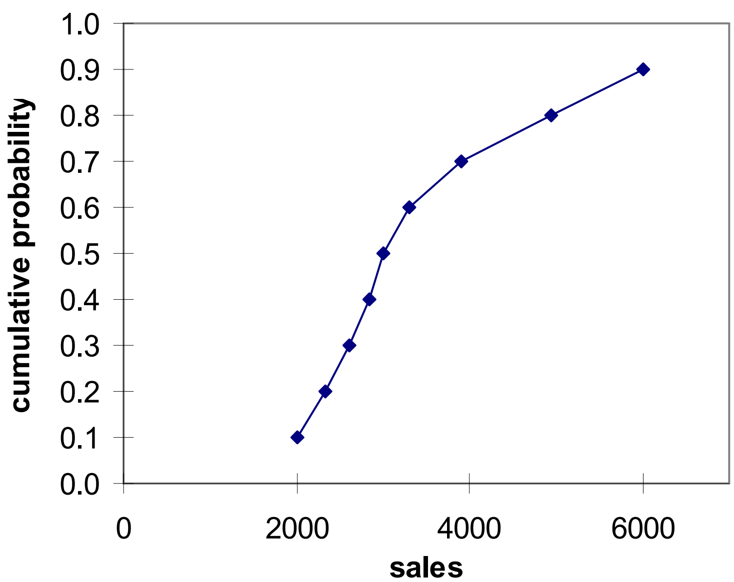
Figure 2



The Bracket Median Method

Of course, in some problems it might be necessary to obtain a more precise approximation of the continuous quantity. We begin by assessing more fractiles to get the cumulative probability distribution shown in **Figure 3**:

Figure 3



Suppose that we are interested in obtaining a five-point approximation of the distribution shown in **Figure 3**. One approach is to divide the distribution into five equally likely brackets, between (A) the .0 and .20 fractiles; (B) the .20 and .40 fractiles; (C) the .40 and .60 fractiles; (D) the .60 and .80 fractiles; and (E) .80 and 1.0 fractiles. The chance that sales fall in any of the brackets is .20. We can approximate bracket A by taking a representative point, say the median of the bracket or the .10 fractile. Similarly, we can approximate (B) by taking the median of that bracket or the .30 fractile.

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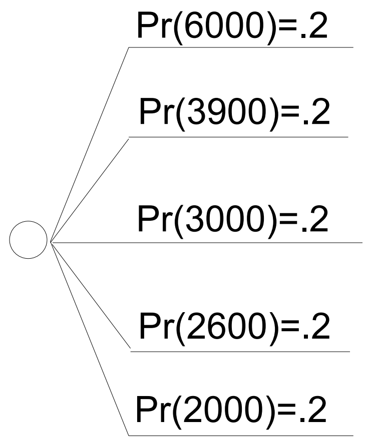
Extending this simple logic to obtain a five point approximation, we use the .10, .30, .50, .70, and .90 fractiles as shown in **Figure 4**. This approach is often called the *bracket median method*.5

Figure 4



[5] To obtain a *n*-point approximation for any *n*, take the 1/2*n*, 3/2*n*, 5/2*n*, …, (2*n*-1)/2*n* fractiles. For *n*=5, we obtain the five fractiles discussed above. For *n*=3, the bracket median method suggests that we assign equal weight to the .17, .50, and .67 fractiles. However, studies show that the weights discussed earlier are better. For a more extended discussion of the bracket median method and other approximation techniques, see Donald L. Keefer and Samuel E. Bodily (1983), "Three-point Approximations for Continuous Random Variables," *Management Science* 29, 595–609, or David E. Bell and Arthur Schleifer, Jr. (1995), *Decision Making under Uncertainty*, Cambridge, MA: Course Technologies, 46–59.

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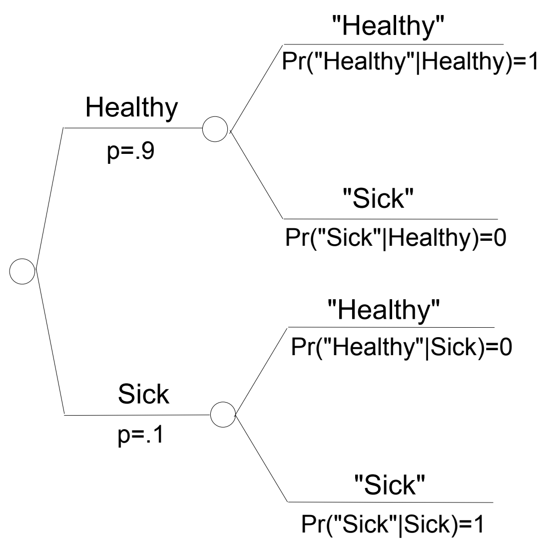
Exhibit 2 Probabilities for Imperfect Information

Information received by a decision maker—experimental results, witness reports, test market data, etc.—is useful in updating judgments about an uncertain world. Generally, this information is not perfect, i.e., it does not completely eliminate the uncertainty about what the future holds in store.

When it comes to information, it is necessary to take into account the quality of the information. But how do you measure quality of information? Suppose that Albert either has or does not have a disease. Although he does not know which is the case, his chance of having the disease is .10 and the probability that he is well is .90. We call these the *prior* probabilities, because they come prior to the collection of information.

Now imagine a perfect test. If he has the disease, the test will tell him that he has the disease. Similarly, he will be informed that he is healthy ("Healthy") if he is indeed healthy. We depict the perfect test in **Figure 5**. The notation, Pr("Healthy" | Healthy), is interpreted in words as "the probability that Albert is informed that he is 'healthy' given that he is in fact healthy."

Figure 5



How should we think about an expert who isn't quite so perfect? First consider a completely uninformative source of information. Imagine a Doctor A who sees many sick and many healthy patients. Doctor A pronounces 80% of the healthy patients "healthy" and 80% of the sick patients "healthy." It is clear that such a doctor is not discriminating and thus provides no information.

Finally, let's understand a partially informative source. Consider a doctor B whose diagnoses are basically correct. If a sick person walks into her office, 90% of the time she pronounces that patient "sick." Furthermore, she diagnoses 95% of healthy patients "healthy." (See **Figure 6**.) We term these the *information quality probabilities*. It turns out that this is the natural way to describe the quality of the information. You should stop to ask yourself why.

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Figure 6

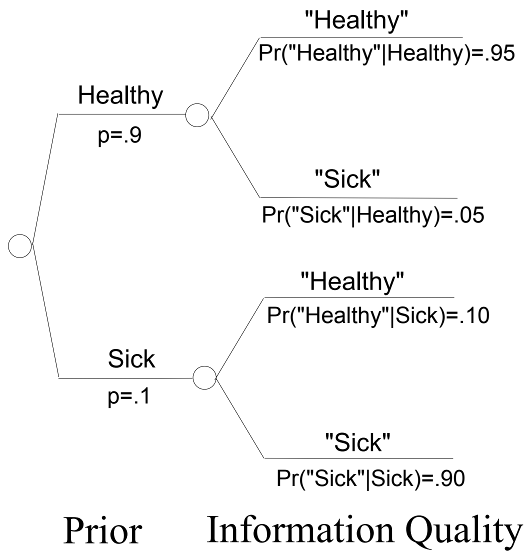
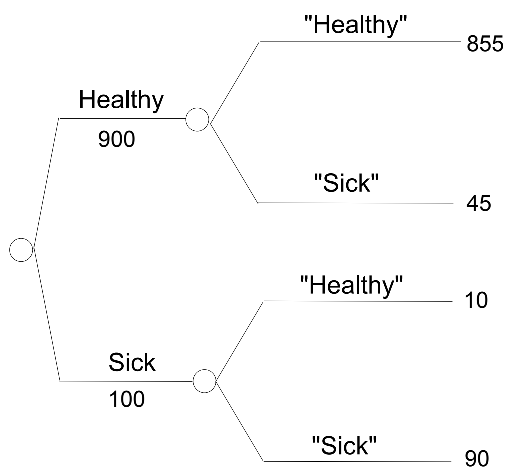


Figure 7



Now let's return to Albert's case. Recall that there's a .90 chance that he is healthy, and a .10 chance that he is sick. He has visited Doctor B, and Doctor B pronounced him "healthy." What is the chance that he is in fact healthy? He could be a healthy person who is diagnosed as healthy, or one of the doctor's mistakes, a sick person who is given good news. Imagine 1,000 people like Albert, 900 of whom are healthy and 100 of whom are sick (**Figure 7**). 95% of the 900 healthy individuals or 855 people will be pronounced "healthy" and 5% will be told they are "sick." Of the 100 sick people, 10 will be told they are "well" and 90 told they are "sick." Thus, a total of 865 people will be pronounced "healthy," 855 of whom are in fact healthy and 10 of whom are sick. Thus, 98.8% of the 865 people who are told they are "healthy" are in fact healthy. A similar calculation (check for yourself!) shows that 67% of the 135 individuals pronounced "sick" are in fact sick (**Figure 8**). These probabilities are often called *posterior probabilities* because they follow information.

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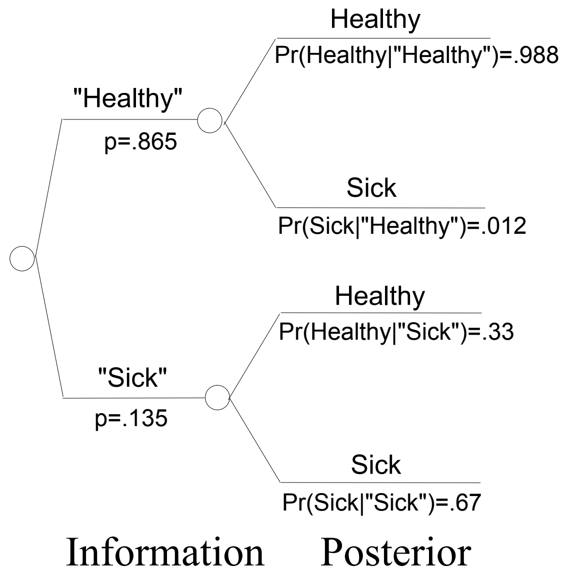
Figure 8



We have used a standard formula from probability theory called *Bayes' Rule*, named after Thomas Bayes, the British Presbyterian minister who discovered the idea in 1763. The procedure is also often called "flipping the tree" because we go from the tree in **Figure 6** to the tree in **Figure 8**. It is important to see that the "conditionals" are flipped. In other words, we start with judgments about sales and information conditional on sales. We end with judgments about information and sales conditional on information.

Both trees are useful. The tree in **Figure 6** is necessary as means of capturing the quality of the information. However, we use the tree in **Figure 8** for evaluating whether it pays to acquire information. Think a moment about the decision sequence. First, we decide to acquire information (go to the doctor), we receive information ("healthy" or "sick"), and then we update our judgments about the uncertain world based on that information.

We apply these basic ideas to the problem that Arrow Cosmetics faces with Marble, leaving the calculations to the reader as an exercise. We begin by describing the information quality. Marble will indicate that the product is either a "Hit" or a "Flop." If the demand is in fact high (600,000 units), Marble will pronounce "Hit" 100% of the time. If the demand is low (200,000 units), Marble will announce "Flop" 5 out of 6 times. If demand is medium (300,000 units), Marble will indicate "Hit" 3 out of 8 times. (See **Figure 9**.) If we flip the tree, following the procedure described above, we get the tree in **Figure 10**.

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Figure 9

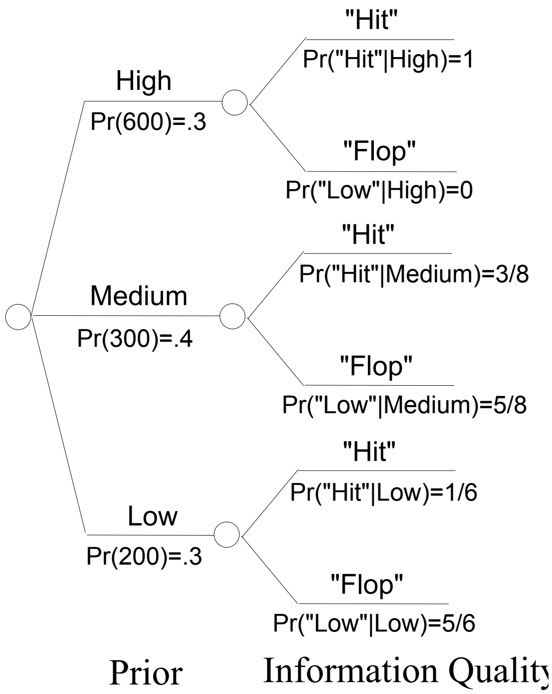
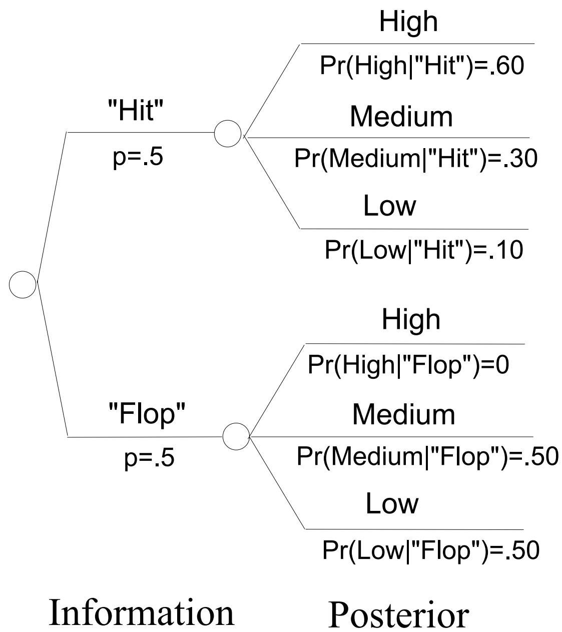


Figure 10



As a final exercise, you may wish to "flip" the tree that describes the information quality of Bargain Research Consortium (**Figure 11**) to arrive at the tree shown in **Figure 12**.

Figure 11

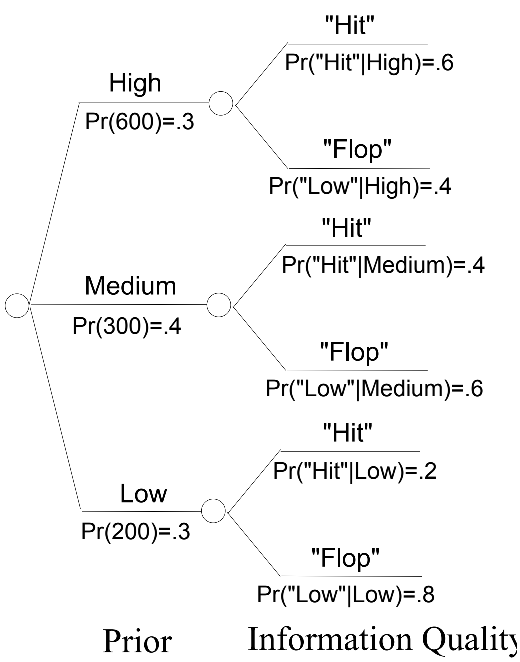
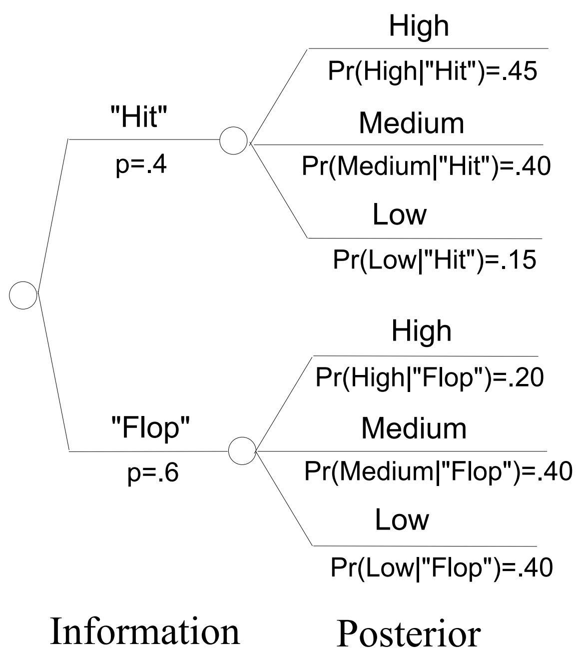


Figure 12



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